Finite-temperature behaviour of a ϕ^6 field system

1982 J. Phys. A: Math. Gen. 15 2231

(http://iopscience.iop.org/0305-4470/15/7/033)

J. Phys. A: Math. Gen. **15** (1982) 2231–2240. Printed in Great Britain

Finite-temperature behaviour of a φ^6 field system

K Babu Joseph and V C Kuriakose
Department of Physics, University of Cochin, Cochin 682022, India

Received 1 September 1981, in final form 22 January 1982

Abstract. The finite-temperature behaviour of a relativistic field with a renormalisable φ^6 self-interaction exhibiting spontaneous symmetry breaking, is studied in one space–one time dimension. Using functional diagrammatic methods, the temperature-dependent effective potential and the critical temperature up to two loops are calculated. The nature of the phase transition is also investigated and is clarified to be one of first order.

1. Introduction

Recently, considerable interest has been shown in the effect of finite temperature on relativistic field theories that exhibit spontaneous symmetry breaking. This is due to the growing conviction among physicists that weak, electromagnetic and strong interactions may owe their origin to spontaneously broken gauge symmetries of a basic Lagrangian. The corresponding Hamiltonian system is in many ways similar to a superconductor. So, arguing by analogy with superconductivity, Kirzhnits and Linde (1972) suggested that a spontaneously broken symmetry in a relativistic field theory coupled to a finite temperature heat bath would be restored above some critical temperature. Later studies (Dolan and Jackiw 1974, Weinberg 1974) have established this fact on a quantitative basis. Functional diagrammatic methods for evaluating effective potentials (Coleman and Weinberg 1973, Iliopoulos *et al* 1975, Jackiw 1974) can be used to study the effect of temperature on a relativistic field system. Dolan and Jackiw (1974) employed this method to evaluate the temperature-dependent effective potential and demonstrated that the symmetry behaviour in φ^4 theory could be restored above a certain temperature.

In this paper we present our calculations on the effect of finite temperature on a model field system exhibiting spontaneous symmetry breaking. We have chosen a general φ^6 field model in $(1+1)$ dimensions such that the classical potential possesses three absolute minima. The Lagrangian enjoys $\varphi \leftrightarrow -\varphi$ internal symmetry, so that the vacuum around any one absolute minimum would correspond to spontaneous symmetry breaking. The model chosen by us has positive mass square and exhibits kink and anti-kink solutions (Lohe 1979). Using lattice approximation and the block-spin renormalisation group method, Boyanovsky and Masperi (1980) have shown that the nature of phase transitions associated with such a field system may be of second order or first order depending on the relative depths of the wells and the inter-site coupling. Besides its importance in particle physics as a model scalar field theory, the φ^6 self-interacting model with a 'specific form' of the potential finds applications in solid state physics also, where it has been used to explain the first-order

0305-4470/82/072231+10\$02.00

phase transition from the ferroelectric to the paraelectric state and the structural phase transitions observed in crystals (Behera and Khare 1980, Lines and Glass 1977, Kittel 1977).

We have employed the functional diagrammatic method to study the temperature effect on the φ^6 field system. The paper is organised in the following way. In § 2 we formulate the effective potential for the model under consideration at zero temperature and show that our model is renormalisable. Section 3 deals with the detailed calculation of the effective potential at finite temperature and the calculations are done up to the two-loop level. It is shown that the broken symmetry originally present in the model can be removed, and the critical temperature is also evaluated in the high-temperature limit. In § 4 we examine the nature of the phase transition. When the system is coupled to a heat bath, the vacuum expectation value $\langle 0|\varphi|0\rangle = \sigma$ is replaced by the thermal average $\langle\varphi\rangle_T = \sigma_T$ taken with respect to a Gibbs ensemble (Kirzhnits and Linde 1976, Linde 1979). The order parameter of the theory thus becomes temperature dependent and vanishes at the critical temperature. The nature of the phase transition is clarified to be one of first order.

2. Evaluation of effective potential at $T = 0$

The model considered by us consists of a real, self-interacting Bose field $\phi(x)$ in $(1+1)$ dimensions, described by the Lagrangian density

$$\mathscr{L}\{\varphi(x)\} = \tfrac{1}{2}(\partial_\mu\varphi)^2 - \tfrac{1}{2}\lambda^2\varphi^2(\varphi^2 - m/\lambda)^2 \tag{1}$$

where $m, \lambda > 0$. It is evident that the above Lagrangian (1) enjoys $\varphi \leftrightarrow -\varphi$ internal symmetry. The classical potential corresponding to this Lagrangian is given by

$$V(\varphi) = \tfrac{1}{2}\lambda^2\varphi^2(\varphi^2 - m/\lambda)^2 \tag{2}$$

such that $V(\varphi) = V(-\varphi)$. The potential has three absolute minima: one at $\varphi = 0$ and the other two at $\varphi = \pm(m/\lambda)^{1/2} = \sigma$. Hence the vacuum around $\varphi = \pm(m/\lambda)^{1/2}$ would correspond to spontaneous symmetry breaking. On shifting the field from $\varphi \to \varphi + \sigma$, where σ is the classical constant scalar field, the Lagrangian (1) becomes

$$\mathscr{L}(\varphi + \sigma) = \tfrac{1}{2}[\partial_\mu(\varphi + \sigma)]^2 - \tfrac{1}{2}\lambda^2(\varphi + \sigma)^2[(\varphi + \sigma)^2 - m/\lambda]^2. \tag{3}$$

The potential in this case is given by

$$V(\varphi + \sigma) = \tfrac{1}{2}\lambda^2\sigma^2(\sigma^2 - m/\lambda)^2 + \tfrac{1}{2}(m^2 - 12\lambda m\sigma^2 + 15\lambda^2\sigma^4)\varphi^2 + \dots. \tag{4}$$

The propagator corresponding to the shifted Lagrangian can be written as

$$\mathrm{i}\Delta^{-1}(\sigma, k) = k^2 - M^2$$

where

$$M^2 = m^2 - 12\lambda m\sigma^2 + 15\lambda^2\sigma^4. \tag{5}$$

The zero-loop (tree approximation) contribution to the effective potential comes from figure 1 and can be written as

$$V_0(\sigma) = \tfrac{1}{2}\lambda^2\sigma^2(\sigma^2 - m/\lambda)^2. \tag{6}$$

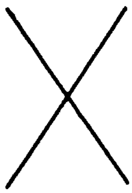

Figure 1. The zero-loop approximation for the effective potential.

The one-loop approximation to the effective potential (figure 2) is given by

$$V_1(\sigma) = -\frac{i}{2}\int \frac{d^2k}{(2\pi)^2} \ln(k^2 + M^2). \tag{7}$$

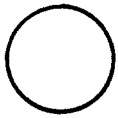

Figure 2. The one-loop approximation for the effective potential.

On rotating this integral into Euclidean space we find

$$V_1(\sigma) = \frac{1}{2}\int \frac{d^2k}{(2\pi)^2} \ln(k^2 + M^2). \tag{8}$$

This integral is ultraviolet divergent; to evaluate it, we cut off the integral at $k^2 = \Lambda^2$, and thus we have

$$V_1(\sigma) = (1/8\pi)M^2 \ln(\Lambda^2/M^2). \tag{9}$$

The divergence in (9) in the lowest order perturbation theory is caused by the graphs shown in figure 3, while there are no infinities associated with the σ^6 term; for instance the graph  is finite.

Hence we may write the effective potential as

$$V(\sigma) = \tfrac{1}{2}\lambda^2\sigma^2(\sigma^2 - m/\lambda)^2 + \tfrac{1}{8}M^2\pi^{-1}\ln(\Lambda^2/M^2) + C_1 + C_2\sigma^2 + C_3\sigma^4. \tag{10}$$

The constants C_1, C_2 and C_3 may be determined by imposing the following normalisation conditions, namely

$$\begin{aligned} &V(\sigma)|_{\sigma=\sqrt{m/\lambda}} = 0;\\ &d^2V(\sigma)/d\sigma^2|_{\sigma=\sqrt{m/\lambda}} = 4m^2;\\ &d^4V(\sigma)/d\sigma^4|_{\sigma=\sqrt{m/\lambda}} = 156\lambda m. \end{aligned} \tag{11}$$

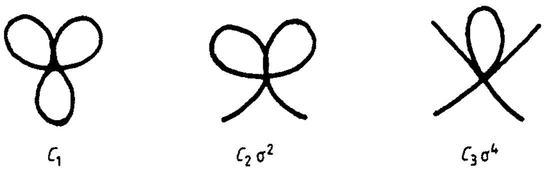

Figure 3. Divergent graphs in the one-loop approximation.

Imposing the conditions (11) on (10), we find that

$$C_1 = -\frac{m^2}{8\pi}\ln\left(\frac{\Lambda^2}{4m^2}\right) - \frac{240}{8\pi}m^2$$

$$C_2 = \frac{12\lambda m}{8\pi}\ln\left(\frac{\Lambda^2}{4m^2}\right) - \frac{777}{16\pi}\lambda m \qquad (12)$$

$$C_3 = -\frac{15\lambda^2}{8\pi}\ln\left(\frac{\Lambda^2}{4m^2}\right) + \frac{1257}{16\pi}\lambda^2.$$

Thus the renormalised effective potential, at zero temperature, for the model chosen by us can be written as

$$V(\sigma) = \tfrac{1}{2}\lambda^2\sigma^2(\sigma^2 - m/\lambda)^2 + \frac{M^2}{8\pi}\ln(4m^2/M^2). \qquad (13)$$

This is the final expression for the effective potential at zero temperature in the one-loop approximation and it does not show any dependence on the cut-off. Since this procedure may be extended straightforwardly to higher loops, the theory is seen to be renormalisable.

3. Effective potential at finite temperature

In this section we may evaluate the effective potential at finite temperature and show that the symmetry breaking present in the model can be removed if the temperature is raised above a certain value called the critical temperature. We may denote the temperature-dependent effective potential by $V^{\mathrm{T}}(\sigma)$. At zero temperature $V^{\mathrm{T}}(\sigma) = V^0(\sigma)$ possesses symmetry breaking solutions. Hence $\partial V^0(\sigma)/\partial\sigma = 0$ for $\sigma \neq 0$. If the finite temperature contribution can eliminate symmetry breaking, then $\partial V^{\mathrm{T}}(\sigma)/\partial\sigma = 2\sigma\,\partial V^{\mathrm{T}}(\sigma)/\partial\sigma^2 = 0$ only if $\sigma = 0$. For large σ^2, $\partial V^{\mathrm{T}}(\sigma)/\partial\sigma^2$ is assumed to be positive. Writing

$$V^{\mathrm{T}}(\sigma) = V^0(\sigma) + \bar{V}^{\mathrm{T}}(\sigma)$$

we have

$$\partial V^0(\sigma)/\partial\sigma^2|_{\sigma=0} + \partial\bar{V}^{\mathrm{T}}/\partial\sigma^2|_{\sigma=0} \geqslant 0.$$

This implies that

$$\partial\bar{V}^{\mathrm{T}}(\sigma)/\partial\sigma^2|_{\sigma=0} \geqslant -\tfrac{1}{2}m^2.$$

Hence the critical temperature can be defined by the relation

$$\partial\bar{V}^{\mathrm{T_c}}(\sigma)/\partial\sigma^2|_{\sigma=0} = -\tfrac{1}{2}m^2. \qquad (14)$$

The effective potential at finite temperature to all loops can be written as (Dolan and Jackiw 1974)

$$V^{\mathrm{T}}(\sigma) = V_0(\sigma) + V_1^{\mathrm{T}}(\sigma) + \mathrm{i}\left\langle \exp\left(\mathrm{i}\int \mathrm{d}^3x\, \mathscr{L}_I(\sigma, \varphi)\right)\right\rangle. \qquad (15)$$

Here $V_0(\sigma)$ is the classical potential—the zero-loop contribution to the effective potential. The zero-loop contribution is temperature independent. $V_1^{\mathrm{T}}(\sigma)$ gives the one-loop approximation. Higher loops are given by $\langle \exp(\mathrm{i}\int \mathrm{d}^3x\, \mathscr{L}_I(\sigma, \varphi))\rangle$—the sum of all the one-particle irreducible vacuum graphs. In our case, the zero-loop contribution to the effective potential is given by

$$V_0(\sigma)=\tfrac{1}{2}\lambda^2\sigma^2(\sigma^2-m/\lambda)^2. \tag{16}$$

Now we may evaluate the contribution from the one-loop to the effective potential at finite temperature. The procedure is to replace the time integral by the sum

$$\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}=\mathrm{i}T\sum_{n=-\infty}^{\infty} \tag{17}$$

where ω is periodic such that $\omega_n=2\pi\mathrm{i}Tn$ (bosons) where $n=0,\pm1,\pm2,\pm3,\ldots$.

Accordingly, (7) may be rewritten as

$$V_1^{\mathrm{T}}(\sigma)=-\frac{\mathrm{i}}{2}\int_k \ln(k^2+M^2)$$

where (18)

$$\int_k=\mathrm{i}T\sum_n\int\frac{\mathrm{d}k}{2\pi}$$

so that

$$\begin{aligned}V_1^{\mathrm{T}}(\sigma)&=\frac{T}{2}\sum_n\int\frac{\mathrm{d}k}{2\pi}\ln(k^2+M^2)\\&=\frac{T}{2}\sum_n\int\frac{\mathrm{d}k}{2\pi}\ln(4\pi^2n^2T^2+E_M^2)\end{aligned} \tag{19}$$

where $E_M^2=k^2+M^2$. The evaluation of (19) is done by performing the summation first. Writing

$$v(E)=\sum_n \ln(4\pi^2n^2T^2+E^2)$$

we have

$$\frac{\mathrm{d}v(E)}{\mathrm{d}E}=\sum_n\left(\frac{2E}{4\pi^2n^2T^2+E^2}\right).$$

Using the identity

$$\sum_{n=-\infty}^{\infty}\frac{1}{x^2+n^2}=(\pi/x)\coth(\pi x) \tag{20}$$

we find that

$$\frac{\mathrm{d}v(E)}{\mathrm{d}E}=\frac{2}{T}\left(\frac{1}{2}+\frac{1}{\mathrm{e}^{E/T}-1}\right).$$

This leads to the result:

$$v(E)=2T^{-1}[\tfrac{1}{2}E+T\ln(1-\mathrm{e}^{-E/T})].$$

Hence (19) becomes

$$V_1^{\mathrm{T}}(\sigma)=\int\frac{\mathrm{d}k}{2\pi}\left[\frac{E_M}{2}+T\ln(1-\mathrm{e}^{-E_M/T})\right] \tag{21}$$

$$=V_1^0(\sigma)+\bar{V}_1^{\mathrm{T}}(\sigma).$$

$V_1^0(\sigma)$ gives the usual zero-temperature one-loop approximation to the effective potential. This may be compared with the expression (7) obtained in § 2: and

$$\bar{V}_1^{\mathrm{T}}(\sigma)=\frac{T}{\pi}\int_0^\infty \mathrm{d}k\ \ln(1-\mathrm{e}^{-E_M/T})$$

gives the temperature-dependent one-loop correction term. Introducing x^2 as

$$x^2T^2=E_M^2-M^2,$$

we have

$$\bar{V}_1^{\mathrm{T}}(\sigma)=\frac{T^2}{\pi}\int_0^\infty \mathrm{d}x\ \ln[1-\exp-(x^2+M^2/T^2)^{1/2}]. \tag{22}$$

This integral may be evaluated by expanding $\bar{V}_1^{\mathrm{T}}$ as a fourier series and in the high-temperature region we find that

$$\bar{V}_1^{\mathrm{T}}(\sigma)=-\tfrac{1}{6}\pi T^2+\tfrac{1}{4}MT. \tag{23}$$

Invoking (14) we can find that the critical temperature correct to one-loop order is

$$T_{\mathrm{c}}=\tfrac{1}{3}m^2/\lambda \tag{24}$$

which is indeed large in the weak-coupling limit.

We shall now proceed to evaluate the two-loop contribution to $V^{\mathrm{T}}(\sigma)$. Our motivation for doing this is to investigate the effect of higher-order loops in determining the critical temperature in a more precise manner. The two-loop contributions come from the two graphs (figures 4(a) and 4(b)). The contribution from figure 4(a) with proper combinatorial factor can be put as

$$V_{2a}^{\mathrm{T}}(\sigma)=\frac{(-24\lambda m)}{8}\left(\int_k\frac{1}{k^2+M^2}\right)^2. \tag{25}$$

Since we are interested only in the temperature-dependent terms, we find that

$$\bar{V}_{2a}^{\mathrm{T}}(\sigma)=(-12\lambda m)[\partial(MT/4)/\partial M^2]^2$$

$$=-\tfrac{3}{16}\lambda mT^2/M^2. \tag{26}$$

The contribution from figure 4(b) can be expressed as

$$V_{2b}^{\mathrm{T}}(\sigma)=\tfrac{1}{12}(24\lambda m)^2\sigma^2(-\mathrm{i})\int_{k_1}\int_{k_2}\int_{k_3}\frac{\delta^2(k_1+k_2+k_3)}{(k_1^2+M^2)(k_2^2+M^2)(k_3^2+M^2)}$$

$$=-48\lambda^2m^2\sigma^2T^3\sum_{n_1}\sum_{n_2}\sum_{n_3}\int\int\int\frac{\mathrm{d}k_1\,\mathrm{d}k_2\,\mathrm{d}k_3}{(2\pi)^3}$$

$$\times\frac{\delta(k_1+k_2+k_3)}{(4\pi^2n_1^2T^2+E_{M_1}^2)(4\pi^2n_2^2T^2+E_{M_2}^2)(4\pi^2n_3^2T^2+E_{M_3}^2)}\delta_{n_1+n_2+n_3}. \tag{27}$$

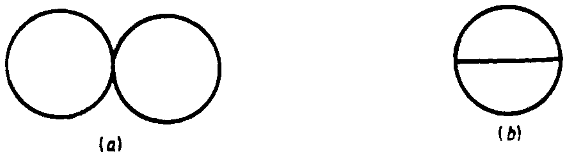

Figure 4. The two-loop approximation for the effective potential.

The summation may be carried out first, and in the higher temperature limit, we find that

$$\bar{V}^{\mathrm{T}}_{2b}(\sigma) = -\frac{96\lambda^2 m^2\sigma^2 T^3}{\pi^2}\int\int\frac{\mathrm{d}k_1\,\mathrm{d}k_2}{(k_1^2+M^2)(k_2^2+M^2)((k_1+k_2)^2+M^2)}. \qquad (28)$$

We may evaluate this integral as follows. Let

$$I = \int\int\frac{\mathrm{d}k_1\,\mathrm{d}k_2}{(k_1^2+M^2)(k_2^2+M^2)[(k_1+k_2)^2+M^2]}.$$

Now define

$$f(x) = \int\frac{\mathrm{d}k\,\mathrm{e}^{\mathrm{i}kx}}{k^2+M^2}$$

as the one-dimensional fourier transform. This integral when evaluated gives:

$$f(x) = \pi\,\mathrm{e}^{-M|x|}/m. \qquad (29)$$

Thus we can find that

$$I = \frac{1}{2\pi}\int\mathrm{d}x\,f^3(x)$$
$$= \frac{1}{\pi}\int_0^\infty \mathrm{d}x\left(\frac{\pi}{M}\mathrm{e}^{-Mx}\right)^3$$

Hence

$$I = \pi^2/3M^4. \qquad (30)$$

Thus we get

$$\bar{V}^{\mathrm{T}}_{2b}(\sigma) = -32\lambda^2 m^2\sigma^2 T^3/M^4. \qquad (31)$$

Hence the temperature-dependent part of the effective potential to the two-loop level is given by

$$\bar{V}^{\mathrm{T}}(\sigma) = -\tfrac{1}{6}\pi T^2 + \tfrac{1}{4}MT - \tfrac{3}{16}\lambda m T^2/M^2 - 32\lambda^2 m^2\sigma^2 T^3/M^4. \qquad (32)$$

The critical temperature is evaluated using relation (14); thus

$$-\tfrac{3}{2}\lambda T_{\mathrm{c}} - \tfrac{9}{4}\lambda^2 T_{\mathrm{c}}^2/m^2 - 32\lambda^2 T_{\mathrm{c}}^3/m^2 = -\tfrac{1}{2}m^2, \qquad (33)$$

which is a cubic equation in T_{c}. Introducing y as

$$T_{\mathrm{c}} = y - \tfrac{3}{128},$$

equation (33) can be cast into the standard form $y^3+py+q=0$ where

$$p=\tfrac{3}{64}m^2/\lambda-1.156\times10^{-4}$$
$$q=-\tfrac{1}{64}m^4/\lambda^2-1.098\times10^{-3}m^2/\lambda+2.574\times10^{-5}. \tag{34}$$

From these relations it is seen that $q^2/4+p^3/27>0$ which is the condition to be satisfied by (33) that it has one real root, so that the critical temperature is determined uniquely. The critical temperature correct to the two-loop level is then given by

$$T_c=[-\tfrac{1}{2}q+(\tfrac{1}{4}q^2+\tfrac{1}{27}p^3)^{1/2}]^{1/3}+[-\tfrac{1}{2}q-(\tfrac{1}{4}q^2+\tfrac{1}{27}p^3)^{1/2}]^{1/3}-\tfrac{3}{128}. \tag{35}$$

This completes our calculation of the critical temperature, correct to two-loop order.

4. Nature of the phase transition

The above calculations reveal that the spontaneous symmetry breaking present in the φ^6 model can be removed by raising the temperature above a critical value. In the language of superconductivity, we may restate this in terms of a phase transition from the ordered phase characterised by $\langle\varphi\rangle\neq0$ to a disordered phase characterised by $\langle\varphi\rangle=0$, as the temperature of the system is raised. We may follow the method of Linde (1979) to study the nature of the phase transition. We replace the vacuum expectation value $\langle0|\varphi|0\rangle=0=\sigma$ by its thermal average $\langle\varphi\rangle_T=\sigma_T$ taken with respect to a Gibbs ensemble so that the order parameter of the theory becomes explicitly temperature-dependent. The ensemble average of the finite-temperature Green function is defined as $\langle\ldots\rangle=\mathrm{Tr}(e^{-H/T}\ldots)/\mathrm{Tr}(e^{-H/T})$ where H is the Hamiltonian governing the system. The parameter characterising the thermodynamic equilibrium state of the φ particles of the system is given by the density of the particles in momentum space:

$$n_k=1/(e^{\omega_k/T}-1)\quad\text{where}\quad n_k=\langle a_k^+a_k\rangle;$$

$\omega_k=(k^2+m^2)^{1/2}$; a_k^+ and a_k are the usual creation and annihilation operators.

The equation of motion corresponding to the Lagrangian (1) is given by

$$\Box\varphi-m^2\varphi+4\lambda m\varphi^3-3\lambda^2\varphi^5=0. \tag{36}$$

On shifting the field from φ to $\varphi+\sigma$ and taking the Gibbs average of the corresponding equation,

$$\begin{aligned}\Box\sigma_T-m^2\sigma_T+4\lambda m\sigma_T^3+12\lambda m\sigma_T\langle\varphi^2\rangle+12\lambda m\sigma_T^2\langle\varphi\rangle\\ -15\lambda^2\sigma_T\langle\varphi^4\rangle-30\lambda^2\sigma_T^2\langle\varphi^3\rangle-30\lambda^2\sigma_T^3\langle\varphi^2\rangle\\ -15\lambda^2\sigma_T^4\langle\varphi\rangle-3\lambda^2\sigma_T^5=0.\end{aligned} \tag{37}$$

Using standard finite-temperature Green-function methods (Abrikosov *et al* 1964) we may find that in the high-temperature limit

$$\begin{aligned}\langle\varphi^2\rangle&=\frac{1}{2\pi}\int_{-\infty}^{\infty}dk(k^2+m^2)^{-1/2}\{\exp[(k^2+m^2)^{1/2}/T]-1\}^{-1}\\&=T/2m.\end{aligned} \tag{38}$$

From similar calculations we also find that

$$\langle\varphi^4\rangle=\tfrac{3}{4}T^2/m^2\qquad\langle\varphi^3\rangle=0\qquad\langle\varphi\rangle=0. \tag{39}$$

Thus (37) becomes

$$\Box\sigma_T - m^2\sigma_T + 4\lambda m\sigma_T^3 + 6\lambda T\sigma_T - 3\lambda^2\sigma_T^5 - 15\lambda^2 T\sigma_T^3/m - \tfrac{45}{4}\lambda^2 T^2\sigma_T/m^2 = 0. \tag{40}$$

Assuming that σ_T is constant we obtain

$$\sigma_T[-m^2 + 4\lambda m\sigma_T^2 + 6\lambda T - 3\lambda^2\sigma_T^4 - 15\lambda^2 T\sigma_T^2/m - \tfrac{45}{4}\lambda^2 T^2/m^2] = 0. \tag{41}$$

This equation has three solutions:

$$\sigma_T = 0 \tag{42}$$

$$\sigma_T^2 = [4\lambda m^2 - 15\lambda^2 T \pm (4\lambda^2 m^4 - 48\lambda^3 m^2 T + 90\lambda^4 T^2)^{1/2}]/6\lambda^2 m. \tag{43}$$

Each solution of these equations defines a possible phase of the field system with its characteristic excitations. On heating the field system from absolute zero, the two branches of σ_T^2 given by (43) can coincide at a temperature $T_1 = 0.10m^2/\lambda$ yielding a common value for σ_T, namely, $\sigma_{T_1}^2 = 5m/12\lambda$. Nevertheless, this is not a phase transition, and as the temperature increases further, the two branches of σ_T^2 will again separate. The existence of the separate branches of σ_T^2 implies that the phase transition at the critical temperature T_c is of first order.

The mass of the excitations may be found by making the shift $\sigma \to \sigma + \delta\sigma$ in (40). Retaining only terms linear in $\delta\sigma$ and using (40)

$$\Box\delta\sigma - [m^2 + 15\lambda^2\sigma_T^4 + 45\lambda^2\sigma_T^2 T/m + \tfrac{45}{4}\lambda^2 T^2/m^2 - 6\lambda T - 12\lambda m\sigma_T^2]\delta\sigma = 0 \tag{44}$$

from which the excitation mass is given by

$$M_\varphi^2 = m^2 - 6\lambda T + \tfrac{45}{4}\lambda^2 T^2/m^2 - 12\lambda m\sigma_T^2 + 45\lambda^2 T\sigma_T^2/m + 15\lambda^2\sigma_T^4. \tag{45}$$

The disordered phase is associated with excitations of mass

$$M_\varphi^2 = m^2 - 6\lambda T + \tfrac{45}{4}\lambda^2 T^2/m^2. \tag{46}$$

However, this mass does not vanish at the critical temperature. The mass obtained from the effective potential $V(\sigma)$, defined by $M^2 = \partial^2 V(\sigma)/\partial\sigma^2|_{\sigma=0}$, can be easily shown to vanish at the transition point. For the disordered phase we find

$$M^2 = m^2 - 3\lambda T - \tfrac{9}{2}\lambda^2 T^2/m^2 + 192\lambda^2 T^3/\pi m^2. \tag{47}$$

The existence of distinct solutions for σ_T as given by (42) and (43) may be indicative of a domain structure of the vacuum. In the case of the Higgs model such a domain structure has been speculated upon (Linde 1979), wherein adjacent domains are associated with opposite signs of σ_T. The domains are separated by kinks, but this is not a stable configuration because they define degenerate minima of the effective potential. The situation is different in the φ^6 case. There is a five-fold multiplicity of values of σ_T which can be associated with different domains in the vacuum. Even though domains carrying condensate values, σ_T, which differ only in sign, may join together due to the collapse of kink walls, there still may be some domains with different absolute values of σ_T. These latter configurations may be assumed to be stable. It is worth mentioning in this context that the existence of a domain wall structure has been very well established experimentally in the case of ferroelectrics which are described by a φ^6 coupled phenomenological model defined in terms of polarisation as the order parameter (Lines and Glass 1977, Kittel 1977).

Acknowledgment

One of us (VCK) is thankful to the University Grants Commission, New Delhi, for the award of a Teacher Fellowship.

References

Abrikosov A A, Gorkov L P and Dzyaloshinski I E 1964 *Methods of Quantum Field Theory in Statistical Physics* (New Jersey: Prentice Hall)
Behera S N and Khare A 1980 *Pramana* **14** 327
Boyanovsky D and Masperi L 1980 *Phys. Rev.* D**21** 1550
Coleman S and Weinberg E 1973 *Phys. Rev.* D**7** 1888
Dolan L and Jackiw R 1974 *Phys. Rev.* D**9** 3320
Iliopoulos J, Itzykson C and Martin A 1975 *Rev. Mod. Phys.* **47** 165
Jackiw R 1974 *Phys. Rev.* D**9** 1686
Kirzhnits D A and Linde A D 1972 *Phys. Lett.* **42**B 471
——1976 *Ann. Phys., NY* **101** 195
Kittel C 1977 *Introduction to Solid State Physics* (Delhi: Wiley Eastern University Edition)
Linde A D 1979 *Rep. Prog. Phys.* **42** 389
Lines M E and Glass A M 1977 *Principles and Applications of Ferroelectrics and Related Materials* (Oxford: Clarendon)
Lohe M A 1979 *Phys. Rev.* D**20** 3120
Weinberg S 1974 *Phys. Rev.* D**9** 3357